Formation of high-gradient regions in freely-decaying and forced two-dimensional hydrodynamic turbulence

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Freely-decaying and forced two-dimensional hydrodynamic turbulence is simulated numerically with a Fourier pseudospectral method using both conventional multiprocessor and graphical clusters. The Euler equation with hyperviscosity is solved to study statistical characteristics of turbulence. In numerical experiments with grid resolution up to 8192 × 8192 a Kraichnan-type turbulence spectrum, $E(k) \sim k^{-3}$ is observed over a few decades in the direct-cascade region of the spectral space. A spatial filtration procedure is used to establish that the main contribution to the spectrum comes from the sharp vorticity gradients in the form of vorticity quasi-shocks. Though the collapse as the process of singularity formation in a finite time is forbidden in two-dimensional hydrodynamics, but there is a strong tendency to the emergence of high-gradient regions. As it follows from our numerical experiments, vorticity gradients increase by more than two orders of magnitude during computation.

Such quasi-singularities are responsible for a strong angular dependence of the spectrum owing to well-localized (in terms of the angle) jets with minor and/or large overlapping. In each jet, the spectrum decreases as k^{-3}, which yields the Kraichnan spectrum after averaging over the angle. The behavior of the third-order structure function accurately agrees with Kraichnan direct cascade concept corresponding to a constant enstrophy flux. It is shown that the power law exponents ζ_n for higher structure functions grow more slowly than the linear dependence of n, which testifies to turbulence intermittency.

Thus, the main result of this investigation is the fact that the Kraichnan-type power-law spectrum is formed owing to quasi-singularities, which appear in solving the Cauchy problem for the two-dimensional Euler equation.

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